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(May 6, 2000)

There are a few obstacles, which bring about imperfect quantum teleportation of a continuous variable state, such as unavailability of maximally entangled two-mode squeezed states, inefficient detection and imperfect unitary transformation at the receiving station. We show that all those obstacles can be understood by a combination of an *asymmetrically-decohered* quantum channel and perfect apparatuses for other operations. For the asymmetrically-decohered quantum channel, we find some counter-intuitive results; one is that teleportation does not necessarily get better as the channel is initially squeezed more and another is when one branch of the quantum channel is unavoidably subject to some imperfect operations, blindly making the other branch as clean as possible may not result in the best teleportation result. We find the optimum strategy to teleport an unknown field for a given environment or for a given initial squeezing of the channel.

PACS number(s); 03.65.Bz, 03.67, 42.50.Dv

Quantum teleportation is one of the important manifestations of quantum mechanics. In particular, quantum teleportation of continuous variable states [1] has attracted a great deal of attention because of a high detection efficiency, handy manipulation of continuous variable states [2–4], and possibility of application to high-quality long-haul telecommunication [6]. Two kinds of protocols have been suggested for continuous variable teleportation; one utilizes the entanglement between quadrature-phase variables [2] and the other between the photon-number sum and the relative phase [5]. Both the protocols employ a squeezed two-mode vacuum for the quantum channel. In this paper, we report how to optimize the quantum teleportation of quadrature-phase variables when the quantum channel and experimental conditions are not perfect.

There are a few obstacles which make the teleportation of quadrature-phase variables imperfect. The perfect quantum teleportation is possible only by a maximally-entangled quantum channel, *i.e.*, by an infinitely squeezed state which is unphysical as it incurs the infinite energy. More over, when the quantum channel is exposed to the real world, it is influenced by the environment, which turns the *pure* squeezed state into a *mixture* and deteriorates the entanglement property. To maximize the channel entanglement, purification protocols for continuous variable states have been suggested by Parker *et al.* [7] for partially-entangled pure states and by Duan *et al.* [8] for mixed Gaussian states. Opartny *et al.* showed that the problem of not having the maximally entangled squeezed vacuum can be overcome by conditional measurements [9]. However, the theoretical suggestions have not been realized by experiment. Further, there are other obstacles in experiment such as imperfect detection efficiency at the sending station and imperfect unitary transformation at the receiving station. We show that the imperfect conditions may be absorbed into the imperfect quantum channel while other apparatuses are treated perfect, and find the optimization condition for the teleportation under a given experimental condition. We show that blindly maximizing the initial entanglement of the quantum channel does not necessarily bring about the best teleportation.

Two modes a and b of the squeezed vacuum are distributed, respectively, to a sending and a receiving station. At the sending station, the original unknown state is entangled with the field mode a of the quantum channel by a 50/50 beam splitter. Two conjugate quadrature variables are measured respectively for the two output fields of the beam splitter using homodyne detectors. Upon receiving the measurement results through the classical channel, the other mode b of the squeezed vacuum is displaced accordingly at the receiving station. Ralph and Lam [3] and Furusawa *et al.* [6] realized quantum teleportation of continuous variable states by experiments.

The quantum teleportation of quadrature-phase variables is completed by the unitary displacement operation at the receiving station. If a field state of its Wigner function $W(\alpha)$ is displaced by β, it is represented by the Wigner function $W(\alpha-\beta)$. In the experiment, the displacement operation is performed using a beam splitter of a high transmittance T [6,3]. To displace a field state of the Wigner function $W(\alpha)$, it is injected into the beam splitter while a high intensity coherent state of amplitude $\sqrt{1-T}\beta$ is injected to the other input port. This operation results in the output field whose Wigner function is [11]

$$W_d(\gamma) = \frac{1}{1-T}\int d^2\alpha W(\alpha) W_{vac}\left(\frac{\gamma-\beta-\sqrt{T}\alpha}{\sqrt{1-T}}\right) \quad (1)$$

where $W_{vac}(\alpha)$ is the Wigner function for the vacuum. We can easily see that *displacing a field by a beam splitter of its transmittance T is equivalent to unitarily-displacing* *the field after it is mixed with the vacuum at a beam splitter of the same T*. Note that mixing a field with the vacuum at a beam splitter results in the same dynamics of the field influenced by the vacuum environment [11].

The inefficient detection at the sending station is another factor which degrades the teleportation [2,10]. When the two photomultimpliers of a homodyne detector have the same efficiency η, the imperfect homodyne detector is described by a perfect homodyne detector with a beam splitter in front [12]. A field passes through the beam splitter of the transmittance η and it is mixed with the vacuum which has been injected into the other input port. The inefficiency of the detection can also be passed to the quantum channel. We will discuss later that inefficiency at the sending station gives an effect not only to the quantum channel but also to the original unknown field to teleport.

We have seen that imperfect operations at the receiving and sending stations can be understood as a combination of the perfect operations with an imperfect mixed quantum channel. Imperfection at the displacement operation is absorbed by the field mode to the receiving station. The field mode to the sending station can absorb inefficiency in the homodyne detection. These considerations lead the quantum channel mixed *asymmetrically* due to a different condition for each mode of the quantum channel. In this paper, we consider that the quantum channel is initially in the two-mode squeezed vacuum each mode of which is separately influenced by a different environment. The study of the teleportation using the *asymmetrically-decohered* quantum channel is important not only because it can explain the experimental situation but also because it gives novel features and deeper understanding of the continuous-variable teleportation. As an application of quantum information, if quantum teleportation is used for a long-haul communication, it is more likely that the two modes of the quantum channel will undergo different environmental conditions. To the best of our knowledge, the impact of the asymmetric channel on the quantum teleportation has not yet been seriously considered.

The quantum channel, which is initially in the two-mode squeezed vacuum of squeezing factor s, is influenced by the thermal environments. It is then represented by the Wigner function [13]

$$W_{ab}(\alpha_a,\alpha_b) = \mathcal{N}\exp\left\{-\frac{2}{m_am_b-c_ac_b}\left[m_a|\alpha_a|^2+m_b|\alpha_b|^2 + \sqrt{c_ac_b}(\alpha_a\alpha_b+\alpha_a^*\alpha_b^*)\right]\right\}, \qquad (2)$$

where $m_i = R_i(1+2\bar{n}_i)+T_i\cosh 2s$ and $c_i = R_i\sinh 2s$ $(i=a,b)$; $\bar{n}_i$ is the average thermal photon number of the environment for the channel mode $i=a,b$. The normalized interaction time $R_i(\equiv 1-T_i)$ is zero when the quantum channel is not subject to the environment and grows to unity when the channel completely assimilates the environment.

It has been shown that a two-mode Gaussian state is separable when a semi-positive well-defined P function can be assigned to it after some local operations [14,15]. The two-mode squeezed state subject to the thermal environment is separable when

$$\left(\frac{T_a}{\bar{n}_aR_a-1}\right)\left(\frac{T_b}{\bar{n}_bR_b-1}\right) \le \coth^2 s \qquad (3)$$

As a special case, if the channel mode b is influenced by the vacuum, *i.e.*, $\bar{n}_b=0$, the channel becomes separable when $R_a \ge 1/(1+\bar{n}_a)$. For this case, it is interesting to note that regardless of initial squeezing of the channel, the separation condition depends only on the average thermal photons influencing the channel mode a, even when the channel is minimally squeezed at the initial instance.

Let us now consider the teleportation using the quantum channel (2). Before any action at the receiving and sending stations, the total state is a product of the original unknown state and the quantum channel. We write the Wigner function $W_o(\alpha)$ for the arbitrary unknown state with use of the quasiprobability P function [18];

$$W_o(\alpha) = \frac{2}{\pi}\int d^2\beta P(\beta)\exp(-2|\alpha-\beta|^2). \qquad (4)$$

Setting homodyne detectors at the two output ports of the beam splitter, quadrature-phase variables p_1 and q_2 are measured at the respective output ports. After the measurement, the Wigner function $W(\alpha_b;q_2,p_1)$ for the field mode b of the quantum channel is conditioned on the measurement results p_1 and q_2. Upon receiving each pair of measurement results, the quadrature variables of the channel field b is displaced accordingly.

If the quantum channel is a pure squeezed vacuum, written as $\sum_n g_n(s)|n\rangle|n\rangle$, after the displacement $\Delta(q_2,p_1)$, Braunstein *et al.* [16] found that the teleportation operator is represented by $\hat{V}\hat{\mathcal{T}}_{bo}$. Here, the transfer operator is $\hat{\mathcal{T}}_{bo} = \sum_n |n\rangle_{bo}\langle n|$ and $\hat{V} = \hat{D}(\sqrt{2}q_2 + i\sqrt{2}p_1)\hat{G}(s)\hat{D}(\Delta(q_2,p_1))$ with the displacement operator $\hat{D}$ and the distortion operator $\hat{G}(s) = \sum_n g_n(s)|n\rangle\ \langle n|$. The teleportation is optimized when the teleportation operator is as close as the unit operator $\hat{\mathbf{1}}$. For the pure squeezed quantum channel, we find that $|\mathrm{Tr}(\hat{V}\cdot\hat{\mathbf{1}})|^2$ is maximized when the displacement is $\Delta(q_2,p_1) = \Delta_o(q_2,p_1) \equiv \sqrt{2}(q_2-ip_1)$. This is in agreement with Braunstein and Kimble [2]. The argument can be extended to the mixed quantum channel (2), which can be written as a Gaussian weighted sum of two-mode coherent states [17]. Using the quantum teleportation operator for the mixed quantum channel, we find that the teleportation is optimized when the displacement $\Delta(q_2,p_1)$ is $\Delta_o(q_2,p_1)$ as for the pure quantum channel.

When the displacement is $\Delta_o(q_2,p_1)$, the Wigner function $W_r(\alpha)$ for the teleported field is

$$W_r(\alpha) = \frac{1/\pi}{n_\tau + \frac{1}{2}} \int d^2\beta P(\beta) \exp\left[-\frac{1}{n_\tau + \frac{1}{2}}|\alpha - \beta|^2\right], \tag{5}$$

where n_τ is a noise factor defined as

$$n_\tau = \frac{1}{4}(\sqrt{T_a} - \sqrt{T_b})^2 \mathrm{e}^{2s} + \frac{1}{4}(\sqrt{T_a} + \sqrt{T_b})^2 \mathrm{e}^{-2s} - \sum_{i=a,b} \frac{1}{2} R_i \bar{n}_i. \tag{6}$$

The fidelity $\mathcal{F}$ is defined as $\mathcal{F} = \pi \int d^2\alpha W_o(\alpha) W_r(\alpha)$ [15]. We note that the fidelity is determined by the channel-dependent noise factor n_τ. When $n_\tau = 1$, the teleportation is perfect with $\mathcal{F} = 1$. The fidelity is zero for $n_\tau \to \infty$. Here, we find an important fact that *the teleportation does not necessarily get better as the quantum channel is initially squeezed more.* When the channel is exposed to the environment for the same duration of time, *i.e.*, $T_a = T_b$, the first term in Eq. (6) becomes zero and the noise is always suppressed as increasing the initial squeezing of the quantum channel. However, when $T_a \neq T_b$, the first term does not vanish and squeezing the quantum channel can degrade the teleportation.

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FIG. 1. The noise factor n_τ defined in Eq. (6) is plotted against the degree of initial squeezing s. The channel mode a is not subject to the environment, *i.e.*, $T_a = 1$ and the channel mode b is subjected to the vacuum environment with the normalized interaction time $R_b = 0.01$ (solid line) and 0.05 (dotted line). The quantum channel which is decohered in the vacuum with normalized interaction time $R_b = 0.01$, corresponds to the *pure* channel for the imperfect displacement with the transmittance $T = 99\%$ as shown in the relation [Eq. (1)].

In Fig. 1, the noise factor n_τ is plotted against initial squeezing s for the asymmetric quantum channel where the channel mode b is influenced by the vacuum environment for the normalized interaction time $R_b = 0.01$ and 0.05 while the channel mode a is not influenced by an environment. The teleportation via the quantum channel, which has been decohered by the vacuum with interaction time $R_b = 0.01$, in fact, corresponds to the teleportation with the *pure* squeezed quantum channel and imperfect displacement using the beam splitter of transmittance $T = 99\%$. It is clearly seen that even for the quantum channel of seemingly-negligible asymmetry, if the channel is initially squeezed more than a certain degree, the teleportation becomes worse.

By the first-derivative of n_τ with regard to s, we find that the teleportation is optimized when the squeezing is

$$\mathrm{e}^{-2s} = \frac{|\sqrt{T_a} - \sqrt{T_b}|}{\sqrt{T_a} + \sqrt{T_b}} \tag{7}$$

for a fixed channel condition T_a and T_b. Note that this result does not depend on the temperature of the environments. Let us assume that for the fixed initial squeezing of the channel the channel mode b is inevitably influenced by an environment of the mean photon number $\bar{n}_b$ for the interaction time T_b. This is to consider an inefficient operation at the receiving station. It is found that the teleportation is optimized by letting the channel mode a interact with the environment for $T_a = T_b \sinh 2s/(\cosh 2s - 2\bar{n}_b - 1)$. We find that when one branch of the channel is inevitably subject to an environment, blindly making the other branch as clean as possible does not bring about the best result. When the two branches are appropriately balanced, we get the best result for the teleportation.

Why can the teleportation get worse while the channel is initially squeezed more? One may *intuitively* expect that the more the quantum channel is squeezed, the better teleportation is. It is true when the channel remains *pure* and not influenced by an environment. However, when the channel is *mixed* by the environment and becomes asymmetric, the conjecture may be wrong. There are many parameters which influences the teleportation. To make the analysis simple without losing the interesting features, we assume for the rest of the paper that the channel is exposed to low-temperature environments only for short periods of time, *i.e.*, $T_i > \bar{n}_i R_i$. In the separation condition (3), $\coth s$ is a monotonous function which gets smaller with increasing s. This means that *the quantum channel remains entangled longer as it is initially squeezed more.* However, as we have seen in Fig. 1, the fidelity of teleportation can be worse with increasing the initial squeezing. Let us write the Wigner function (2) to highlight the EPR correlation [19] of the quantum channel as follows

$$W_{ab}(\alpha_a, \alpha_b) \approx \mathcal{N} \exp\Big(\frac{-2}{n_a n_b - c_a c_b} \times \Big[\mathrm{e}^{2s}\{(q'_a + q'_b)^2 + (p'_a - p'_b)^2\} + \mathrm{e}^{-2s}\{(q'_a - q'_b)^2 + (p'_a + p'_b)^2\}\Big]\Big) \tag{8}$$

where $q'_{a,b} = \sqrt{T_{b,a}} q_{a,b}$ and $p'_{a,b} = \sqrt{T_{b,a}} p_{a,b}$. This result has been obtained assuming the short interaction time with low-temperature environments, again. As the initial squeezing $s \to \infty$, the Wigner function $W_{ab} \to C\delta(\sqrt{T_b} q_a + \sqrt{T_a} q_b)\delta(\sqrt{T_b} p_a - \sqrt{T_a} p_b)$. It is clear that the asymmetric channel has the EPR correlation between the scaled quadrature variables; positions q'_a and q'_b and momenta p'_a and p'_b. This is somewhat similar to the relation between an original phase space and amplified or dissipated phase space depending on the scale factor T_a/T_b. When the channel is strongly squeezed the channel entanglement teleports the original photon to the scaled space, which brings about large noise in the teleported state. For a large squeezing, even though the quantum channel is strongly entangled, the strong entanglement results in inefficient teleportation because the entanglement is between the scaled spaces.

An imperfect detection efficiency at the receiving station can also be analyzed as a combination of perfect detection with a beam splitter in front. At the beam splitter, not only the channel state but also the unknown original field are mixed with the vacuum. The teleported field is like Eq. (5) with the exponential function modified; $\exp[-|\alpha-\eta\beta|^2/(n_\tau+\frac{1}{2})] \to \exp[-|\alpha-\eta\sqrt{T_a}\beta|^2/(n_\tau+\frac{1}{2})]$, where T_a is the detection efficiency in this case. The average fidelity is maximized for the displacement factor $\eta = 1/\sqrt{T_a}$. Assuming $\eta = 1/\sqrt{T_a}$ and the channel mode a interacting with the vacuum for the period R_a we find the noise factor $n_\tau = \mathrm{e}^{-2s} + R_a/T_a$ with use of Eq.(6). This shows that the teleportation is more efficient when the channel is initially squeezed more, which is in agreement with the earlier result [2].

We have shown that the important experimental errors can be absorbed by an imperfect mixed quantum channel while the experimental operations are assumed to be perfect. Because the experimental error does not occur symmetrically between the receiving and sending stations, it is important to study the influence of the asymmetrically-decohered quantum channel on the teleportation. When the teleportation is applied for long-haul communication, it is not possible to put the quantum-channel generator at the center of the receiving and sending stations so it is natural to imagine that the quantum channel is in general subject to different environmental conditions. For the asymmetrically-decohered quantum channel we found that the strong initial squeezing does not always optimize the teleportation because the asymmetric quantum channel has the EPR correlation between the scaled phase spaces. We found the value of initial squeezing which optimized the teleportation.

We know that when $n_\tau \geq 1$, any nonclassical properties which may be implicit in the original unknown state can be washed out [10,15]. It has been found that when the quantum channel is symmetrically-decohered, $T_a = T_b$ and $\bar{n}_a = \bar{n}_b$, separability of the quantum channel coincides with disabling a transfer of any nonclassical features [15]. On the other hand, when the channel is asymmetrically-decohered, even an entangled mixed channel may not transfer any nonclassical property of the original state.

ACKNOWLEDGMENTS

We thank Mr. H. Jeong for stimulating discussions. This work was supported in part by the Brain Korea project (D-0055) of the Korean Ministry of Education.

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